

*Center for Advanced Studies in  
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*CASMA Technical Note*

*Number 1*

**Estimated Standard Error of a Mean  
When There are only two Observations**

*Robert L. Brennan<sup>†</sup>*

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<sup>†</sup>Robert L. Brennan is E. F. Lindquist Chair in Measurement and Testing and Director, Center for Advanced Studies in Measurement and Assessment (CASMA), 210D Lindquist Center, College of Education, University of Iowa, Iowa City, IA 52242 (email: robert-brennan@uiowa.edu).

Center for Advanced Studies in  
Measurement and Assessment (CASMA)  
College of Education  
University of Iowa  
Iowa City, IA 52242  
Tel: 319-335-5439  
Web: [www.education.uiowa.edu/casma](http://www.education.uiowa.edu/casma)

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**Theorem:** When  $n = 2$ , the estimated standard error of the mean is

$$\hat{\sigma}_{\bar{X}} = \frac{|X_1 - X_2|}{2}.$$

**Proof:** The well-know formula for the estimated standard error of a mean is

$$\hat{\sigma}_{\bar{X}} = \frac{\hat{\sigma}_X}{\sqrt{n}}. \quad (1)$$

When  $n = 2$ , an unbiased estimate of the variance of the individual scores is:

$$\begin{aligned} \hat{\sigma}_X^2 &= \sum_{i=1}^2 (X_i - \bar{X})^2 \\ &= (X_1 - \bar{X})^2 + (X_2 - \bar{X})^2 \\ &= \left(X_1 - \frac{X_1 + X_2}{2}\right)^2 + \left(X_2 - \frac{X_1 + X_2}{2}\right)^2 \\ &= \left(\frac{X_1 - X_2}{2}\right)^2 + \left(\frac{X_2 - X_1}{2}\right)^2 \\ &= 2 \left(\frac{X_1 - X_2}{2}\right)^2 \\ &= \frac{(X_1 - X_2)^2}{2}, \end{aligned}$$

which means that the estimated standard deviation of the two scores is

$$\hat{\sigma}_X = \frac{|X_1 - X_2|}{\sqrt{2}}. \quad (2)$$

Replacing Equation 2 in Equation 1 gives

$$\hat{\sigma}_{\bar{X}} = \frac{|X_1 - X_2|}{\sqrt{2}\sqrt{2}} = \frac{|X_1 - X_2|}{2}.$$

*QED*

**Lemma:** Under normality assumptions, when  $n = 2$ , a 68% confidence interval for the mean is

- $(X_2, X_1)$  if  $X_1 > X_2$  or
- $(X_1, X_2)$  if  $X_1 < X_2$ .

**Proof:** If  $X_1 > X_2$ , then a 68% confidence interval for the mean extends from

$$\bar{X} - \hat{\sigma}_{\bar{X}} = \bar{X} - \frac{X_1 - X_2}{2} = \frac{X_1 + X_2}{2} - \frac{X_1 - X_2}{2} = X_2$$

to

$$\bar{X} + \hat{\sigma}_{\bar{X}} = \bar{X} + \frac{X_1 - X_2}{2} = \frac{X_1 + X_2}{2} + \frac{X_1 - X_2}{2} = X_1.$$

The proof of the second part of the lemma is similar.

*QED*

**Note:** These proofs were originally provided by the author for use in the Achievement Levels Settings projects conducted by ACT, Inc. in the 1990s (see Reckase, 2000). In that context, the two scores,  $X_1$  and  $X_2$ , were themselves means (over items and raters) for two groups of raters each of which independently evaluated half of the items in a standard setting process, and the resulting estimated standard error was for the mean of these means.

## References

Reckase, M. D. (2000). *The evolution of the NAEP Achievement Levels Setting Process: A summary of the research and development efforts conducted by ACT*. Iowa City, IA: Act, Inc.