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Manual for LEGS Version 2.0

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Disclaimer of Warranty

No warranties are made, express or implied, that LEGS is free of error, that it is consistent with any particular standard, or that it will meet the requirements of any particular application. The author disclaims any direct or consequential damages resulting from use of this program.

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Abstract

LEGS (*L*inking with *E*quivalent *G*roups or *S*ingle Group Design) is an ANSI C computer program for linking scores on two tests using various statistical methods including mean, linear, porallel-linear, and equipercentile methods with and without postsmoothing. Also provided are a number of subgroup-invariance statistics for such linked scores. These statistics are discussed in Chapter 10 of Kolen and Brennan (2004). Both Macintosh and PC/Windows versions of LEGS are available.

Introduction

LEGS (*L*inking with *E*quivalent *G*roups or *S*ingle Group Design) is an ANSI C computer program for linking scores on two tests using various statistical methods including mean, linear, parallel-linear, and equipercentile methods with and without postsmoothing. Also provided are a number of subgroup-invariance statistics for such linked scores. These statistics are described in detail in Chapter 10 of Kolen and Brennan (2004), which uses the same verbal and notational conventions as in LEGS. LEGS produces five primary output files. Both Macintosh and PC/Windows versions of LEGS are available.

To execute LEGS, the user double-clicks the LEGS icon. LEGS then prompts the user for

- the type of input (frequency distributions or raw scores) and
- the name of the file containing the control cards.

The file containing the control cards must be in the same folder as the LEGS application, or the full pathname for the control cards must be specified. After the user types a return, LEGS executes. When execution is complete (usually only a second or two), the message "Successful execution" is printed in the same window used to specify the name of the control cards file.

In the text of this manual (not necessarily the tables and figures), variables are put in italics, and file names are put in quotes. If a variable is the name of a file, italics are used.

Frequency Distributions as Input

When frequency distributions are used as input, a run of LEGS requires a file containing a set of control cards, and two files containing frequency distributions. These files should be in text-only format.

Control Cards

Table 1 provides an illustrative set of control cards when frequency distributions are used as input. There are nine control cards; i.e., the file should contain nine lines. The control cards must be in the order discussed below. The first three lines in Table 1 (column identifiers and a blank line) should *not* be in the file.

For each control card, all parameters are separated from each other by any number of spaces and/or tabs. Unless otherwise specified, the order in which parameters are provided is fixed, and the parameters are integers. LEGS looks for a linebreak (newline or return) character at the end of each line, which is generated by typing a return.[1] Note that the linebreak produced by hitting the return key generates different ASCII code under Macintosh and PC/Windows operating systems.[2] Therefore, a control cards file generated using a Macintosh computer will not work as input for a PC/Windows computer.

[1] Strictly speaking, each control card should be terminated by any uninterrupted sequence of newline and/or return characters.

[2] Actually, there are differences among Macintosh, DOS, and Unix systems.

Table 1: Illustrative Control Cards with Frequency Distributions as Input

```
COLUMNS  11111111112
12345678901234567890

sc_ACT
sc_ITED
2  MALE FEMALE
manual-yfreqs
manual-xfreqs
.672 .659 .689
.5   2 .30 1.00
1 36 1
1
```

Card 1: Y Scores

yid alphanumeric identifier (with no spaces) for Y (maximum of 20 characters).

LEGS provides linking results relative to Y in the sense that Y equivalents of X scores are provided along with relevant summary statistics.

Card 2: X Scores

xid alphanumeric identifier (with no spaces) for X (maximum of 20 characters).

Card 3: Subgroups

nsg number of subgroups ($nsg \leq 9$).

sgid [∗] *nsg* alphanumeric subgroup identifiers, with no spaces within each identifier (maximum of 8 characters).

There are two ways to get linking equivalents when there are no subgroups:

- Set $nsg = 0$, and do not provide an *sgid* [0] identifier. In this case, the frequency files should have a column of scores and a single column of frequencies.

- Set *nsg* = 1, and provide an *sgid*[1] identifier. In this case, the frequency files should have a column of scores followed by two identical columns of frequencies.

Obviously, when there are no subgroups, much of the output in LEGS is irrelevant.

Card 4: File for Y Frequencies

yfilename alphanumeric name of file that contains Y scores and frequencies.

This file must be in the same folder as the LEGS application, or the full pathname must be specified.

Card 5: File for X Frequencies

xfilename alphanumeric name of file that contains X scores and frequencies. (This may *not* be the same as *yfilename*.)

This file must be in the same folder as the LEGS application, or the full pathname must be specified.

Card 6: Correlations

corr[0] correlation for the combined group. If *corr*[0] = −99, then the randomly equivalent groups design is assumed; otherwise, the single group design is assumed.

corr[i] correlation for the *nsg* subgroups. These correlations should be set to −99 for the randomly equivalent groups design.

If these correlations are set to −99, then the *rmsel* output (discussed later) is not provided, and the standard errors used in cubic spline postsmoothing are for the randomly equivalent groups design (Lord, 1982, pp. 167–169, the so-called "Discrete Case, Two Groups"). If these correlations are present, (i.e., not equal to −99), the standard errors used in cubic spline postsmoothing are for the single group design under normality assumptions (Lord, 1982, pp. 169–171, the so-called "Continuous Case, One Group").

Card 7: Equipercentile Smoothing

slim interpolate if percentile rank is outside the range [*slim*, 1-*slim*]. *slim* should be a floating point number between 0 and 100. In accordance with the conventions in Kolen and Brennan (1995, 2004), almost always *slim* = .5.

kn number of smoothing values.

s [∗] *kn* smoothing values (floating point numbers between 0 and 1).

Card 8: Truncation

validYL lowest valid score for Y.

validYH highest valid score for Y.

truncation $truncation = 0 \Rightarrow$ no truncation.
$truncation = 1 \Rightarrow$ truncate rounded scores only.
$truncation = 2 \Rightarrow$ truncate both rounded and unrounded scores.
Note that rounding is done before truncation.

Card 9: Create Equating Error Input File

eeinput If $eeinput \neq 0$, create input file for the computer program Equating Error (Hanson, 2000).

This card must be present even if the Equating Error input file is not requested.

Frequency Files

The frequencies for Y and X need to be provided in separate files. These files should be in text-only format. In Kolen and Brennan (2004) Y and X designate raw scores (almost always number-of-items correct); here, Y and X usually refer to scale scores.[3] LEGS requires that these scale scores be integers (not necessarily positive integers).

Figure 1 provides the frequencies for Y and X for the illustrative run of LEGS. In accordance with the specifications in the control cards in Table 1, the frequencies for Y should be in a file named "manual-yfreqs" and the frequencies for X should be in a file named "manual-xfreqs". For each file, the entries in each line are:

- score (integer),
- frequency for combined group,
- frequency for first subgroup,

  ⋮
- frequency for last subgroup.

[The line numbers in Figure 1 should *not* be included in the file(s).] The scores must be integers ordered from low to high, and the entries must be separated by one or more blanks and/or tabs. When a particular score has no frequency (e.g., a score of 164 for X), it is not necessary to include that score.

Each line must end with a linebreak (newline or return) character, which is generated by typing a return. Note that the linebreak produced by hitting the

[3] Raw scores can always be interpreted as scale scores resulting from a linear transformation of raw scores with an intercept of 0 and a slope of 1.

Line	Y Frequencies	X Frequencies
1	9 3 0 3	163 1 1 0
2	10 6 3 3	169 2 2 0
3	11 20 7 13	173 2 1 1
4	12 18 5 13	177 1 0 1
5	13 46 18 28	181 6 3 3
6	14 90 40 50	186 8 4 4
7	15 158 63 95	192 15 9 6
8	16 338 113 225	199 23 16 7
9	17 454 159 295	207 38 27 11
10	18 412 155 257	216 48 29 19
11	19 738 293 445	225 56 34 22
12	20 811 300 511	234 70 43 27
13	21 914 366 548	242 109 50 59
14	22 748 302 446	249 88 41 47
15	23 838 382 456	255 89 42 47
16	24 802 374 428	260 111 44 67
17	25 399 198 201	264 116 51 65
18	26 522 251 271	268 145 56 89
19	27 336 183 153	272 148 68 80
20	28 263 135 128	275 171 67 104
21	29 287 153 134	278 189 67 122
22	30 169 98 71	282 193 79 114
23	31 76 51 25	285 179 65 114
24	32 53 28 25	288 241 95 146
25	33 17 12 5	290 248 100 148
26	34 66 42 24	293 279 111 168
27	35 9 8 1	297 267 109 158
28	36 35 27 8	301 309 114 195
29		305 319 116 203
30		309 342 121 221
31		314 388 157 231
32		319 369 155 214
33		323 372 155 217
34		328 399 159 240
35		333 437 158 279
36		337 411 186 225
37		342 416 186 230
38		347 408 190 218
39		351 363 175 188
40		355 357 174 183
41		360 299 150 149
42		364 236 131 105
43		368 187 122 65
44		372 121 66 55
45		377 42 29 13
46		382 10 8 2

Figure 1: Frequencies for Y and X in separate files.

Table 2: Illustrative Control Cards with Raw Scores as Input

```
COLUMNS  1111111111222222222
123456789012345678901234567

sc_ACT      1
sc_ITED     2
2  MALE  FEMALE  3766  4862
manual-raw
.5   2 .30 1.00
1 36 1
1
```

return key generates different ASCII code under Macintosh and PC/Windows operating systems. Therefore, frequency files generated using a Macintosh computer will not work as input for a PC/Windows computer.

Raw Data as Input

When raw data are used as input, a run of LEGS requires a file containing a set of control cards, and a single file containing the raw data (i.e., Y and X scores for each examinee). These files should be in text-only format.

Control Cards

Table 2 provides an illustrative set of control cards when raw data are used as input. There are seven control cards; i.e., the file should contain seven lines in the order discussed below. The first three lines in Table 2 (column identifiers and a blank line) should *not* be in the file.

For each control card, all parameters are separated from each other by any number of spaces and/or tabs. Unless otherwise specified, the order in which parameters are provided is fixed, and the parameters are integers. LEGS looks for a linebreak (newline or return) character at the end of each line, which is generated by typing a return.[4] Note that the linebreak produced by hitting the return key generates different ASCII code under Macintosh and PC/Windows operating systems.[5] Therefore, a control cards file generated using a Macintosh computer will not work as input for a PC/Windows computer.

[4]Strictly speaking, each control card should be terminated by any uninterrupted sequence of newline and/or return characters.

[5]Actually, there are differences among Macintosh, DOS, and Unix systems.

Card 1: Y Scores

yid	alphanumeric identifier (with no spaces) for Y (maximum of 20 characters).
yscol	column for reading Y scores
lowY	lowest score for Y (optional)
highY	highest score for Y (optional)

Both *lowY* and *highY* must be present or both must be absent. LEGS provides linking results relative to Y in the sense that Y equivalents of X scores are provided along with relevant summary statistics.

Card 2: X Scores

xid	alphanumeric identifier (with no spaces) for X (maximum of 20 characters).
xscol	column for reading X scores
lowX	lowest score for X (optional)
highX	highest score for X (optional)

Both *lowX* and *highX* must be present or both must be absent.

Card 3: Subgroups

nsg	number of subgroups ($nsg \leq 9$).
sgid [*]	*nsg* alphanumeric subgroup identifiers, with no spaces within each identifier (maximum of 8 characters).
nsub [*]	*nsg* integers indicating the numbers of examinees in each subgroup.

The ordering of the numbers of examinees in subgroups must correspond with the names of the subgroups.

If there are no subgroups, do *not* set $nsg = 0$. Rather, set $nsg = 1$, provide an identifier for *sgid* [1], and set *nsub* [1] to the total group sample size. Obviously, when there are no subgroups, much of the output is irrelevant.

Card 4: File for Raw Data

bivfilename	alphanumeric name of file that contains raw data.

This file must be in the same folder as the LEGS application, or the full pathname must be specified.

Card 5: Equipercentile Smoothing

slim	interpolate if percentile rank is outside the range [*slim*, 1-*slim*]. *slim* should be a floating point number between 0 and 100. In accordance with the conventions in Kolen and Brennan (1995, 2004), almost always *slim* = .5.
kn	number of smoothing values.
$s\,[*]$	*kn* smoothing values (floating point numbers between 0 and 1).

Card 6: Truncation

validYL	lowest valid score for Y.
validYH	highest valid score for Y.
truncation	$truncation = 0 \Rightarrow$ no truncation. $truncation = 1 \Rightarrow$ truncate rounded scores only. $truncation = 2 \Rightarrow$ truncate both rounded and unrounded scores.

Card 7: Create Equating Error Input File

eeinput	If $eeinput \neq 0$, create input file for the computer program Equating Error (Hanson, 2000).

This card must be present even if the Equating Error input file is not requested.

Raw Data File

Each record of the raw data file must contain a Y score for an examinee in column *yscol* and an X score for the examinee in column *xscol*. (It is not necessary that $xscol > yscol$.) Columns must be separated by at least one space and/or tab. The order of the records in the file must be: *nsub*[*1*] records for examinees in subgroup 1, *nsub*[*2*] records for examinees in subgroup 2, ..., *nsub*[*nsg*] records for examinees in subgroup *nsg*.

As an example, consider again the control cards in Table 2. The first two cards specify that the Y scores are in column 1 and the X scores are in column 2. The third card specifies that the first 3766 records are for males, and the next 4862 records are for females.

Output

LEGS generates five output files. Letting "cc" (without the quotes) be a generic designator for the control cards, these output files are identified as:

- "cc-yout",
- "cc-xout",
- "cc-out",
- "cc-unr-equiv", and
- "cc-r-equiv"

(without the quotes).[6] For example, if the control cards in Table 1 were in a file called "manualf", then the first output file would be named "manualf-yout", the second would be "manualf-xout", etc. Each of these output files is described next and illustrated in the Tables 3–13 on pages 16–26. In each of these tables the filename is in parentheses in the table caption, under the assumption that the control cards file name is "manualf".

"cc-yout" and "cc-xout"

For both Y and X, LEGS provides separate files that list the

1. scores,
2. frequencies,
3. probabilities,
4. cumulative probabilities, and
5. percentile ranks.

This information is provided for the combined group and all subgroups, as illustrated in Tables 3 and 4. The scores are consecutive integers beginning with the lowest score in *yfilename* (for Y) and *xfilename* (for X) and ending with the highest scores in these files. Scores with zero frequencies between the lowest and highest scores are reported, as illustrated in Tables 4.

"cc-out"

This file is separated into five parts:

1. information from/about control cards and data files—basically an echoing of the information provided in the control cards (e.g., Table 5);

[6]In addition, if *eeinput* $\neq 0$, an additional file named "cc-eeinput" is created that can be used as input for the computer program Equating Error (Hanson, 2000).

2. summary statistics for Y and X, effect sizes, standardized mean differences, linear conversions of X to Y, and parallel-linear conversions of X to Y (e.g., Table 6);

3. unrounded moments and pairwise group difference statistics (e.g., Table 7);

4. rounded moments and pairwise group difference statistics (e.g., Table 8);

5. *REMSD*, *ewREMSD*, and *rmsel* statistics (e.g., Table 9).

Most of the information provided in this file is self-explanatory, or is discussed fully by Kolen and Brennan (2004, chap. 10). The following paragraphs provide a bit more detail about some aspects of the output.

The illustrated output in Tables 5–9 is for the case in which frequency distributions are used as input. When raw data are used as input for the single group design, correlations are computed by LEGS and reported in "cc-out", and *rmsel* values are computed for equipercentile conversions and reported in "cc-out".

Effect Sizes. Effect sizes are expressed in terms of pooled standard deviation units. So, for example, if males (M) and females (F) are the two subgroups, then the effect size is

$$ES = \frac{\overline{X}_M - \overline{X}_F}{\sqrt{\dfrac{SS_M + SS_F}{n_M + n_F - 2}}},$$

where SS stands for sums of squares, and n stands for sample size. LEGS provides effect sizes for all combinations of pairs of subgroups.

Truncation. It is possible for a Y equivalent to be outside the range of valid Y scores. For example, valid scores for the the ACT Assessment (Y in the illustrative control cards in Table 1) range from 1–36 (inclusive). If the user specifies that *truncation* is 1, then rounded scores below 1 will be reported as 1 and rounded scores above 36 will be reported as 36 in the file "cc-r-equiv" (discussed in the next section). If the user specifies that *truncation* is 2, then both rounded and unrounded scores scores will be truncated in the files "cc-r-equiv" and "cc-unr-equiv" (discussed in the next section).

Note that truncation can affect the values of the *REMSD* and *ewREMSD* statistics. This is especially likely for the mean method, somewhat likely for the linear and parallel linear methods, and much less likely for the equipercentile methods.

Interpolation. Linear interpolation is applied only to smoothed—more specifically, cubic-spline postsmoothed—equipercentile equivalents that are outside the PR range [*slim*, 100 − *slim*]. Since *slim* is usually set to .5 (given the conventions discussed by Kolen & Brennan, 1995, 2004), linear interpolation usually occurs for scores associated with *PRs* outside the range [.5, 99.5].

Cubic-spline postsmoothing is essentially a regression procedure and, therefore, does not produce symmetric results. Consequently, a three-step, somewhat ad hoc, procedure is used to obtain the "final" postsmoothed equipercentile equating results:

1. the equipercentile procedure with postsmoothing is used for converting X scores to the scale of Y for those X scores that have *PRs* in the range $[slim,\ 100 - slim]$;

2. the equipercentile procedure with postsmoothing is used for converting Y scores to the scale of X for those Y scores that have *PRs* in the range $[slim,\ 100 - slim]$;

3. the results in the previous two steps are averaged.

The last step is considerably more complicated than it may appear because the desired results are X scores converted to the scale of Y. These Y equivalents are available directly from the first step, but obtaining them from the second step requires finding the inverse function of Y to X, which is accomplished in LEGS using the Newton-Raphson procedure.

From the above discussion, it is clear that linear interpolation may need to be applied for the X to Y equivalents (Step 1) as well as the Y to X equivalents (Step 2). However, we illustrate the process here for the X to Y equivalents (Step 1) only.

Consider, again, the Y and X scores and frequencies in Figure 1. Let the Y scores be designated $y_1 \dots y_J$, which is 9–36 for the illustrative data. Similarly, let the X scores be designated $x_1 \dots x_I$, which is 163–382 for the illustrative data.[7,8] For Y, the lower limit for the interpolation is set at $y_1 - .5$ and the upper limit is $y_J + .5$. Similarly, for X the lower limit for the interpolation is set at $x_1 - .5$ and the upper limit is $x_I + .5$.

Now, consider the *PRs* for X for the combined group (0) in Table 4.[9] For Step 1, postsmoothed equipercentile Y equivalents are obtained by LEGS for X scores in the range $[199, 376]$, which we designate generically as $[slimxl, slimxh]$. Linear interpolation is used outside that range. For the low end of the range, linear interpolation occurs between the coordinates

$$(x - 1 - .5, y - 1 - .5) \quad \text{and} \quad (x - slimxl, e - Y(slimxl)),$$

where $e - Y(slimxl)$ is the postsmoothed equipercentile equivalent for $x = slimxl$ converted to the scale of Y. For the illustrative data, linear interpolation at the

[7] In the early chapters of Kolen and Brennan (2004), K designates the number of items in a test. Here, I designates the total number of integer scale score points for X, which need not be K.

[8] There are I consecutive positive integers for X, and J consecutive positive integers for Y, which should not be confused with the number of non-zero frequencies for X and Y scores, respectively. For the illustrative data, as indicated in Figure 1, there are 46 X scores with non-zero frequency, but there are $I = 382 - 163 + 1 = 220$ potential X scores.

[9] As indicated in the top of the table, there are 36 scores with percentile ranks smaller than $slim = .5$, which means that linear interpolation will be used for scores at or below $163 + 36 - 1 = 198$. Similarly, since there are six scores with percentile ranks larger than $100 - slim = 99.5$, linear interpolation will be used for scores at or above $382 - 6 + 1 = 377$.

low end of the range occurs between

$$(162.5, 8.5) \quad \text{and} \quad (199, e - Y(199)).$$

Similarly, at the high end of the range, linear interpolation occurs between

$$(x - \mathit{slimxh}, e - Y(\mathit{slimxh})) \quad \text{and} \quad (x - I + .5, y - J + .5)$$

For the illustrative data, interpolation at the high end occurs between

$$(376, e - Y(376)) \quad \text{and} \quad (382.5, 36.5).$$

When the input is frequency distributions, LEGS uses the lowest and highest scores in the frequency files (see Figure 1) as a basis for determining the lower and upper limits for linear interpolation. So, for the illustrative example, since the lowest score in the Y frequencies file is 9, the lower limit for Y is set to 8.5. This may or may not be acceptable to the user. In the case of the ACT Assessment, because the lowest reported score is 1, the user might want the lower limit for linear interpolation to be $1 - .5 = .5$. If so, the user could set up the Y freqencies file as follows:

```
1 0 0 0
2 0 0 0
3 0 0 0
4 0 0 0
5 0 0 0
6 0 0 0
7 0 0 0
8 0 0 0
9 3 0 3
.
.
.
36 35 27 8
```

Or, more simply, the user could set up the file as

```
1 0 0 0
9 3 0 3
.
.
.
36 35 27 8
```

That is, the user can control the lower and upper limits for linear interpolation by prepending or appending one or more scores with zero frequencies.

When raw scores are used as input, the user can control the lower and upper limits for linear interpolation by providing explicit values for lowY and highY in Card 1, and/or lowX and highX in Card 2. So, for example, if the user set $\mathit{lowY} = 1$ and $\mathit{highY} = 36$, the lower limit for linear interpolation for Y would be $1 - .5 = .5$.

“cc-unr-equiv” and “cc-r-equiv”

Unrounded Y equivalents for each of the linking methods are provided in the file “cc-unr-equiv”, which also lists points of intersection for all possible pairs of groups (combined and subgroups). For the illustrative example, selected equivalents are provided in Table 10, and selected points of intersection are provided in Table 11.

There are $(nsg + 2) * (5 + kn) + nsg + 1$ columns in the first part of the “cc-unr-equiv” file, where *nsg* is the number of subgroups, and *kn* is the number of smoothing values. In sequence, these columns are:

- X scores,
- *nsg* + *1* frequencies,
- *nsg* + *1* mean equivalents,
- *nsg* + *1* linear equivalents,
- *nsg* + *1* parallel linear equivalents,
- *nsg* + *1* unsmoothed equipercentile equivalents,
- *nsg* + *1* equipercentile postsmoothed equivalents for the first smoothing value,
- ...
- *nsg* + *1* equipercentile postsmoothed equivalents for the *kn*-th smoothing value,
- $RMSD(x)$ values for each of the $4 + kn$ linking procedures,
- *nsg* + *1* standard errors for unsmoothed equipercentile equivalents.

Because of space constraints, sets of these columns are listed vertically in Table 10. So, for example, for $x = 319$ for the combined group, the equipercentile Y equivalent with a smoothing value of .30 is 22.00309, and for $x = 320$ it is 22.15970.

The standard errors are computed as indicated next.

- For the randomly equivalent groups design with frequency distributions as input (correlations set to -99), see the “discrete case, two groups” in Lord (1982).
- For the single group design with frequency distributions as input (correlations provided), see the “continuous case, one group” in Lord (1982).
- For the single group design with raw data as input, see the “discrete case, one group” in Lord (1982).

The points of intersection for pairs of groups are provided in the manner indicated in Table 11 for the illustrative example. (To save space intersection points are not provided in Table 11 for smoothed equipercentile equating.) Consider the equipercentile (no smoothing) results for males vs. females in Table 11:

```
MALE vs. FEMALE:      +  378( 36.20, 36.25)  -
```

This means that for X scores below 378, males have *higher* (+) Y equivalents than for females; but for X scores at or above 378, males have *lower* (−) Y equivalents than for females. At $x = 378$, the Y equivalent for males is 36.20, and for females it is 36.25. Recall that reported ACT scores are integers in the 1–36 range, but the results reported here are for unrounded equivalents, which can be outside the 1–36 range.

Rounded Y equivalents for each of the linking methods, as well as $RMSD(x)$ values, are provided in the file "cc-r-equiv", which also lists points of intersection for all possible pairs of groups (combined and subgroups). For the illustrative example, selected equivalents and $RMSD(x)$ values are provided in Table 12, and selected points of intersection are provided in Table 13. Standard errors are not provided in the "cc-r-equiv" file.

Other Issues

LEGS makes use of several functions in preparation, Teukolsky, Vetterling, and Flannery (1992). The cubic-spline smoothing algorithm used in LEGS is described by Reinsch (1967).

As noted previously, it is especially important that the control cards and frequencies files use the type of linebreak appropriate to the application. That is, when LEGS is used with a Macintosh, the linebreaks should be the Macintosh type, and when LEGS is used with a PC/Windows operating system, the linebreaks should be the DOS type.

References

Hanson, B. A. (2000) Equating Error: A Program for Computing Equating Error Using the Bootstrap [Computer softward and manual]. Available from www.uiowa.edu/~casma.

Kolen, M. J., & Brennan, R. L. (1995). *Test equating: Methods and Practices.* New York: Springer-Verlag.

Kolen, M. J., & Brennan, R. L. (2004). *Test equating, scaling, and linking: Methods and Practices* (2nd ed.). New York: Springer-Verlag.

Lord, F. M. (1982). The standard error of equipercentile equating. *Journal of Educational Statistics, 7*, 165–174.

Press, W. H., Teukolsky, S. A., Vetterling, W. T., & Flannery, B. P. (1992). *Numerical recipes in C* (2nd ed.). New York: Cambridge University Press.

Reinsch, C. H. (1967). Smoothing by spline functions. *Numerische Mathematik, 10*, 177–183.

Appendix: Output Files

Table 3: Frequencies, Cumulative Frequencies, and Percentile Ranks for Y ("manualf-yout")

Y = sc_ACT for Group = COMBINED

Number of y scores such that Q(y) < slim (0.500) = 4
Number of y scores such that Q(y) > 1-slim (99.500) = 2

y	freq	cum freq	g(y)	G(y)	Q(y)
9	3	3	0.00035	0.00035	0.017
10	6	9	0.00070	0.00104	0.070
11	20	29	0.00232	0.00336	0.220
12	18	47	0.00209	0.00545	0.440
13	46	93	0.00533	0.01078	0.811
14	90	183	0.01043	0.02121	1.599
15	158	341	0.01831	0.03952	3.037
16	338	679	0.03917	0.07870	5.911
17	454	1133	0.05262	0.13132	10.501
18	412	1545	0.04775	0.17907	15.519
19	738	2283	0.08554	0.26460	22.184
20	811	3094	0.09400	0.35860	31.160
21	914	4008	0.10593	0.46453	41.157
22	748	4756	0.08669	0.55123	50.788
23	838	5594	0.09713	0.64835	59.979
24	802	6396	0.09295	0.74131	69.483
25	399	6795	0.04624	0.78755	76.443
26	522	7317	0.06050	0.84805	81.780
27	336	7653	0.03894	0.88700	86.752
28	263	7916	0.03048	0.91748	90.224
29	287	8203	0.03326	0.95074	93.411
30	169	8372	0.01959	0.97033	96.054
31	76	8448	0.00881	0.97914	97.473
32	53	8501	0.00614	0.98528	98.221
33	17	8518	0.00197	0.98725	98.627
34	66	8584	0.00765	0.99490	99.108
35	9	8593	0.00104	0.99594	99.542
36	35	8628	0.00406	1.00000	99.797

Y = sc_ACT for Group = MALE

.
.
.

Y = sc_ACT for Group = FEMALE

.
.
.

Table 4: Frequencies, Cumulative Frequencies, and Percentile Ranks for X (“manualf-xout”)

```
X = sc_ITED for Group = COMBINED

Number of x scores such that P(x) < slim (0.500) = 36
Number of x scores such that P(x) > 1-slim (99.500) = 6

    x      freq  cum freq     f(x)      F(x)      P(x)

  163        1         1   0.00012   0.00012     0.006
  164        0         1   0.00000   0.00012     0.012
  165        0         1   0.00000   0.00012     0.012
  166        0         1   0.00000   0.00012     0.012
  167        0         1   0.00000   0.00012     0.012
  168        0         1   0.00000   0.00012     0.012
  169        2         3   0.00023   0.00035     0.023
  170        0         3   0.00000   0.00035     0.035
  171        0         3   0.00000   0.00035     0.035
  172        0         3   0.00000   0.00035     0.035
  173        2         5   0.00023   0.00058     0.046
  174        0         5   0.00000   0.00058     0.058
    .
    .
    .
  371        0      8455   0.00000   0.97995    97.995
  372      121      8576   0.01402   0.99397    98.696
  373        0      8576   0.00000   0.99397    99.397
  374        0      8576   0.00000   0.99397    99.397
  375        0      8576   0.00000   0.99397    99.397
  376        0      8576   0.00000   0.99397    99.397
  377       42      8618   0.00487   0.99884    99.641
  378        0      8618   0.00000   0.99884    99.884
  379        0      8618   0.00000   0.99884    99.884
  380        0      8618   0.00000   0.99884    99.884
  381        0      8618   0.00000   0.99884    99.884
  382       10      8628   0.00116   1.00000    99.942


X = sc_ITED for Group = MALE

    .
    .
    .


X = sc_ITED for Group = FEMALE

    .
    .
    .
```

Table 5: Input and Data ("manualf-out")

```
INFORMATION FROM/ABOUT CONTROL CARDS AND DATA FILES

Input Type:  Frequency Distributions

Name of control cards file: manualf

Listing of control cards:

sc_ACT
sc_ITED
2  MALE FEMALE
manual-yfreqs
manual-xfreqs
.672 .659 .689
.5   2 .30 1.00
1 36 1
1


Interpretation of control cards:

Filename containing Y scores and frequencies: manual-yfreqs
   ID for Y: sc_ACT
   Number of scores in file: 28
   Lowest score in file: 9
   Highest score in file: 36
   Column for for Y scores: 1
   Column for combined group frequency: 2
   Columns for subgroup frequencies: 3  4
   Names of subgroups: MALE  FEMALE

Filename containing X scores and frequencies: manual-xfreqs
   ID for X: sc_ITED
   Number of scores in file: 46
   Lowest score in file: 163
   Highest score in file: 382
   Column for for X scores: 1
   Column for combined group frequency: 2
   Columns for subgroup frequencies: 3  4
   Names of subgroups: MALE  FEMALE

Correlations:  0.67200  0.65900  0.68900

For smoothed equipercentile equivalents, linear interpolation
    occurs outside the PR range [0.500,99.500]
The 2 smoothing values are:  0.300  1.000

Truncation values:
   Lowest value of Y reported is 1
   Highest value of Y reported is 36
Truncation values are applied to rounded equivalents only
  (truncation = 1).

Equating error input file has been generated.
```

Table 6: Summary Statistics, Effect Sizes, Standardized Mean Differences, and Parameters for Linear and Parallel Linear Conversions ("manualf-out")

```
Summary Statistics for Y = sc_ACT

      GROUP     N      w WT     MEAN      SD       SKEW      KURT

COMBINED(0)  8628   1.00000  22.19692   4.21832   0.34974   3.21350
    MALE(1)  3766   0.43649  22.83404   4.40107   0.31327   3.10862
  FEMALE(2)  4862   0.56351  21.70341   4.00201   0.32539   3.23797


Summary Statistics for X = sc_ITED

      GROUP     N      w WT     MEAN      SD       SKEW      KURT

COMBINED(0)  8628   1.00000 314.19089  36.18574  -0.64871   3.30241
    MALE(1)  3766   0.43649 315.49973  39.12991  -0.75718   3.29010
  FEMALE(2)  4862   0.56351 313.17709  33.69416  -0.53880   3.22329


            Effect Size   Effect Size
Subgroups      for Y         for X      Difference

  1 - 2       0.27046       0.06423       0.20623


           Standardized  Standardized
             Mean Diff     Mean Diff
Subgroups      for Y         for X      Difference

  1 - 2       0.26803       0.06419       0.20384


Estimates of parameters for linear conversion of X to Y

   GROUP     intercept     slope

COMBINED     -14.42959   0.11657
    MALE     -12.65127   0.11247
  FEMALE     -15.49409   0.11877

Estimates of parameters for parallel linear conversion of X to Y

   GROUP     intercept     slope

COMBINED     -14.42959   0.11657
    MALE     -13.94504   0.11657
  FEMALE     -14.80491   0.11657
```

Table 7: Unrounded Moments and Pairwise Group Difference Statistics ("manualf-out")

		Using Unrounded Equivalents			
Subgroups	Method	MEAN	SD	SKEW	KURT
0	Data	22.19692	4.21832	0.34974	3.21350
	Mean	22.19692	36.18574	-0.64871	3.30242
	Linear	22.19692	4.21832	-0.64871	3.30242
	P-Linear	22.19692	4.21832	-0.64871	3.30242
	Equi	22.19663	4.22605	0.35541	3.23088
	Equi(0.30)	22.18723	4.20021	0.33008	3.14306
	Equi(1.00)	22.18774	4.20147	0.33309	3.14865
1	Data	22.83404	4.40107	0.31327	3.10862
	Mean	22.83404	39.12991	-0.75718	3.29011
	Linear	22.83404	4.40107	-0.75718	3.29011
	P-Linear	22.83404	4.56153	-0.75718	3.29011
	Equi	22.83419	4.40674	0.31636	3.12375
	Equi(0.30)	22.83260	4.40671	0.29276	3.09781
	Equi(1.00)	22.83269	4.40731	0.29069	3.08575
2	Data	21.70341	4.00201	0.32539	3.23797
	Mean	21.70341	33.69416	-0.53880	3.22330
	Linear	21.70341	4.00201	-0.53880	3.22330
	P-Linear	21.70341	3.92786	-0.53880	3.22330
	Equi	21.69861	3.99347	0.29629	3.15151
	Equi(0.30)	21.67417	3.96299	0.29902	3.13291
	Equi(1.00)	21.67225	3.95864	0.29098	3.10757

		Using Unrounded Equivalents			
Subgroups	Method	MD	ewMD	MAD	ewMAD
1 - 0	Mean	-0.67172	-0.67172	0.67172	0.67172
	Linear	0.48828	0.66088	0.48828	0.66088
	P-Linear	0.48455	0.48455	0.48455	0.48455
	Equi	0.35493	0.52050	0.35494	0.52093
	Equi(0.30)	0.36403	0.45511	0.36403	0.45574
	Equi(1.00)	0.36260	0.45141	0.36353	0.45387
2 - 0	Mean	0.52030	0.52030	0.52030	0.52030
	Linear	-0.37389	-0.46483	0.37389	0.46483
	P-Linear	-0.37532	-0.37532	0.37532	0.37532
	Equi	-0.27605	-0.45391	0.27608	0.45537
	Equi(0.30)	-0.29228	-0.44743	0.29228	0.44767
	Equi(1.00)	-0.29458	-0.45712	0.29507	0.45753
1 - 2	Mean	-1.19202	-1.19202	1.19202	1.19202
	Linear	0.86300	1.12571	0.86300	1.12571
	P-Linear	0.85987	0.85986	0.85987	0.85986
	Equi	0.63474	0.97441	0.63480	0.97631
	Equi(0.30)	0.65969	0.90254	0.65969	0.90254
	Equi(1.00)	0.66057	0.90853	0.66057	0.90853

Table 8: Rounded Moments and Pairwise Group Difference Statistics ("manualf-out")

		Using Rounded Equivalents			
Subgroups	Method	MEAN	SD	SKEW	KURT
0	Data	22.19692	4.21832	0.34974	3.21350
	Mean	20.79045	15.41039	-0.26097	1.27584
	Linear	22.17293	4.18011	-0.62457	3.27941
	P-Linear	22.17293	4.18011	-0.62457	3.27941
	Equi	22.15635	4.19658	0.30272	3.19693
	Equi(0.30)	22.13108	4.21805	0.34559	3.14093
	Equi(1.00)	22.13108	4.21805	0.34559	3.14093
1	Data	22.83404	4.40107	0.31327	3.10862
	Mean	21.24881	15.51834	-0.32105	1.28712
	Linear	22.83909	4.35922	-0.73292	3.22920
	P-Linear	22.80563	4.57201	-0.81917	3.39798
	Equi	22.81147	4.40677	0.36242	3.23520
	Equi(0.30)	22.79766	4.38677	0.32643	3.15981
	Equi(1.00)	22.78014	4.34376	0.27479	3.04924
2	Data	21.70341	4.00201	0.32539	3.23797
	Mean	20.54258	15.23691	-0.22927	1.28068
	Linear	21.69128	4.04580	-0.53023	3.23238
	P-Linear	21.66002	3.98772	-0.51086	3.28380
	Equi	21.60325	3.91376	0.29478	3.16066
	Equi(0.30)	21.58309	3.92672	0.26527	3.05689
	Equi(1.00)	21.58309	3.92672	0.26527	3.05689

		Using Rounded Equivalents			
Subgroups	Method	MD	ewMD	MAD	ewMAD
1 - 0	Mean	-0.31071	-0.15909	0.31071	0.15909
	Linear	0.51686	0.65909	0.51686	0.65909
	P-Linear	0.48653	0.47727	0.48653	0.47727
	Equi	0.37236	0.47727	0.37236	0.47727
	Equi(0.30)	0.38373	0.43636	0.38373	0.43636
	Equi(1.00)	0.36865	0.43636	0.36865	0.43636
2 - 0	Mean	0.31520	0.15909	0.31520	0.15909
	Linear	-0.36153	-0.46818	0.36153	0.46818
	P-Linear	-0.39392	-0.38182	0.39392	0.38182
	Equi	-0.33454	-0.47727	0.33454	0.47727
	Equi(0.30)	-0.32632	-0.45455	0.32632	0.45455
	Equi(1.00)	-0.32632	-0.46818	0.32632	0.46818
1 - 2	Mean	-0.62703	-0.31818	0.62703	0.31818
	Linear	0.87668	1.12727	0.87668	1.12727
	P-Linear	0.88074	0.85909	0.88074	0.85909
	Equi	0.70700	0.95455	0.70700	0.95455
	Equi(0.30)	0.71743	0.89091	0.71743	0.89091
	Equi(1.00)	0.70341	0.90455	0.70341	0.90455

Table 9: *REMSD* and *ewREMSD* for Unrounded and Rounded Equivalents; *rmsel* Statistics ("manualf-out")

	Mean	Linear	Par Lin	Equi	s=0.30	s=1.00
Unrounded						
REMSD	0.14015	0.10500	0.10109	0.08719	0.08894	0.08921
ewREMSD	0.14015	0.14085	0.10109	0.13447	0.12496	0.12633
Rounded						
REMSD	0.13244	0.15564	0.15618	0.14085	0.13904	0.13748
ewREMSD	0.09455	0.17605	0.15427	0.16445	0.15843	0.16068

Unrounded rmsel values

GROUP	Mean	Linear	Par Lin
COMBINED(0)	33.49701	3.41658	3.41658
MALE(1)	36.38052	3.63455	3.70369
FEMALE(2)	31.07245	3.15627	3.12777

Table 10: Unrounded Equivalents ("manualf-unr-equiv")

x	f(x)_0	f(x)_1	f(x)_2
	Mean_0	Mean_1	Mean_2
	Lin_0	Lin_1	Lin_2
	PLin_0	PLin_1	PLin_2
	Equi_0	Equi_1	Equi_2
	0.30_0	0.30_1	0.30_2
	1.00_0	1.00_1	1.00_2
	Rx_Mean	Rx_Lin	Rx_Plin
	Rx_Equi	Rx_0.30	Rx_1.00
	SE_0	SE_1	SE_2
	.		
	.		
	.		
319	0.04277	0.04116	0.04401
	27.00603	26.33431	27.52633
	22.75753	23.22773	22.39503
	22.75753	23.24208	22.38221
	22.00468	22.41887	21.72422
	22.00309	22.49097	21.72779
	22.00024	22.48845	21.73256
	0.14015	0.09790	0.10109
	0.08185	0.09077	0.09009
	0.06397	0.10569	0.07928
320	0.00000	0.00000	0.00000
	28.00603	27.33431	28.52633
	22.87411	23.34020	22.51380
	22.87411	23.35866	22.49879
	22.25134	22.63874	21.96413
	22.15970	22.62687	21.85428
	22.12963	22.59378	21.82606
	0.14015	0.09716	0.10109
	0.07933	0.09115	0.09057
	0.06388	0.08363	0.07903
	.		
	.		
	.		

Table 11: Unrounded Equivalents—Intersections (“manualf-unr-equiv”)

```
Intersection Points for Unrounded Scores
     Using Format [sign x(y_focal,y_comparison) sign ...] where
     "sign" is the sign of the difference score (focal-comparison),
     "x" is the x-score associated with a change in the sign of the difference,
     "y_focal" is the corresponding y-score for the focal group,
     "y_comparison" is the corresponding y-score for the comparison group

Method   Focal Group vs. Comparison Group

Mean

         COMBINED vs. MALE:    +
         COMBINED vs. FEMALE:  -
         MALE vs. FEMALE:      -

Linear

         COMBINED vs. MALE:    -
         COMBINED vs. FEMALE:  +
         MALE vs. FEMALE:      +

Parallel Linear

         COMBINED vs. MALE:    -
         COMBINED vs. FEMALE:  +
         MALE vs. FEMALE:      +

Equipercentile

         COMBINED vs. MALE:    -  378( 36.21, 36.20)  +
         COMBINED vs. FEMALE:  +  378( 36.21, 36.25)  -
         MALE vs. FEMALE:      +  378( 36.20, 36.25)  -

Equipercentile with s=0.30

Equipercentile with s=1.00
```

Table 12: Rounded Equivalents ("manualf-r-equiv")

```
  x     f(x)_0    f(x)_1    f(x)_2
        Mean_0    Mean_1    Mean_2
         Lin_0     Lin_1     Lin_2
        PLin_0    PLin_1    PLin_2
        Equi_0    Equi_1    Equi_2
        0.30_0    0.30_1    0.30_2
        1.00_0    1.00_1    1.00_2
       Rx_Mean    Rx_Lin   Rx_Plin
       Rx_Equi   Rx_0.30   Rx_1.00

         .
         .
         .
319    0.04277   0.04116   0.04401
      27        26        28
      23        23        22
      23        23        22
      22        22        22
      22        22        22
      22        22        22
       0.23706   0.17796   0.17796
       0.00000   0.00000   0.00000

320    0.00000   0.00000   0.00000
      28        27        29
      23        23        23
      23        23        22
      22        23        22
      22        23        22
      22        23        22
       0.23706   0.00000   0.17796
       0.15662   0.15662   0.15662


      .
      .
      .
```

Table 13: Rounded Equivalents—Intersections (“manualf-r-equiv”)

```
Intersection Points for Unrounded Scores
     Using Format [sign x(y_focal,y_comparison) sign ...] where
     "sign" is the sign of the difference score (focal-comparison),
     "x" is the x-score associated with a change in the sign of the difference,
     "y_focal" is the corresponding y-score for the focal group,
     "y_comparison" is the corresponding y-score for the comparison group

Method   Focal Group vs. Comparison Group

Mean

         COMBINED vs. MALE:    -
         COMBINED vs. FEMALE:  -
         MALE vs. FEMALE:      -

Linear

         COMBINED vs. MALE:    -
         COMBINED vs. FEMALE:  +
         MALE vs. FEMALE:      +

Parallel Linear

         COMBINED vs. MALE:    -
         COMBINED vs. FEMALE:  +
         MALE vs. FEMALE:      +

Equipercentile

         COMBINED vs. MALE:    -
         COMBINED vs. FEMALE:  -
         MALE vs. FEMALE:      +

Equipercentile with s=0.30

Equipercentile with s=1.00
```